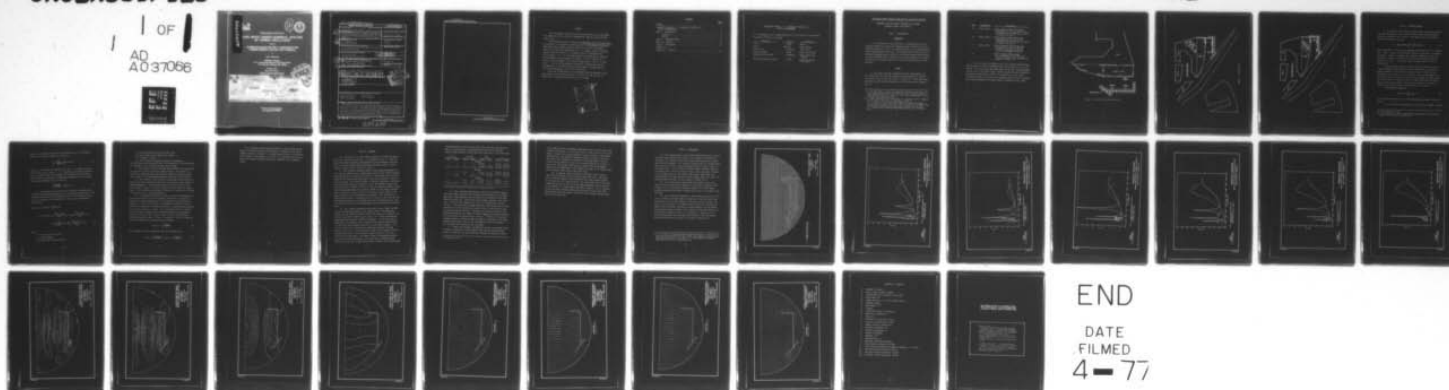AD-A037 066   ARMY ENGINEER WATERWAYS EXPERIMENT STATION   VICKSBURG MISS   F/G 13/2
LONG BEACH HARBOR NUMERICAL ANALYSIS OF HARBOR OSCILLATIONS. RE--ETC(U)
FEB 77   J J WANSTRATH

UNCLASSIFIED

1 OF 1
AD
A037066

NL

END
DATE
FILMED
4-77

MISCELLANEOUS PAPER H-76-20

LONG BEACH HARBOR NUMERICAL ANALYSIS OF HARBOR OSCILLATIONS

Report 4

ALTERNATE PLANS FOR PIER J COMPLETION AND TANKER TERMINAL PROJECT (NO LANDFILL)
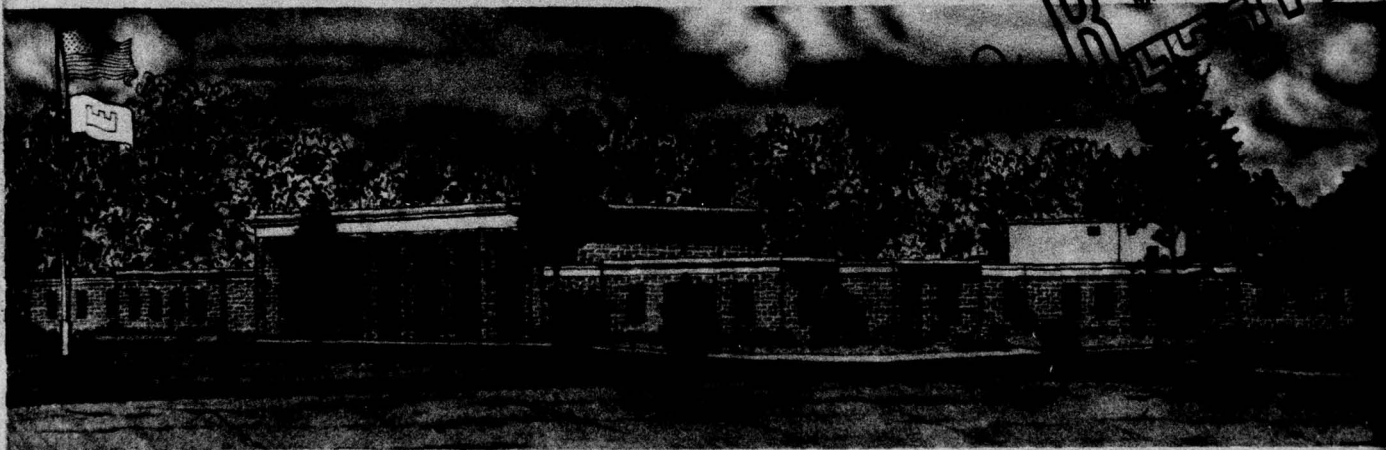
by

John J. Wanstrath

Hydraulics Laboratory
U. S. Army Engineer Waterways Experiment Station
P. O. Box 631, Vicksburg, Miss. 39180

February 1977
Report 4 of a Series

Approved For Public Release; Distribution Unlimited

DDC
MAR 17 1977

Prepared for Port of Long Beach
Long Beach, Calif. 90801

SECURITY CLASSIFICATION OF THIS PAGE *(When Data Entered)*

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER Miscellaneous Paper H-76-20	**2. GOVT ACCESSION NO.**	**3. RECIPIENT'S CATALOG NUMBER**
4. TITLE (and Subtitle) LONG BEACH HARBOR NUMERICAL ANALYSIS OF HARBOR OSCILLATIONS. Report 4. ALTERNATE PLANS FOR PIER J COMPLETION AND TANKER TERMINAL PROJECT (NO LANDFILL).		**5. TYPE OF REPORT & PERIOD COVERED** Report 4 of a series
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) John J. Wanstrath		**8. CONTRACT OR GRANT NUMBER(s)** WES 76-4
9. PERFORMING ORGANIZATION NAME AND ADDRESS U. S. Army Engineer Waterways Experiment Station Hydraulics Laboratory P. O. Box 631, Vicksburg, Mississippi 39180		**10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS**
11. CONTROLLING OFFICE NAME AND ADDRESS Port of Long Beach Long Beach, California 90801		**12. REPORT DATE** February 1977
		13. NUMBER OF PAGES 32
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office) *Miscellaneous paper Sep 75 – Oct 76*		**15. SECURITY CLASS. (of this report)** Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

WES-MP-H-76-20-4

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

34 p.

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Harbor oscillations   Piers (docks)
Long Beach Harbor   Tanker terminals
Numerical analysis

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A hybrid finite element numerical model was used to calculate harbor resonance for the Pier J completion and tanker terminal project of Long Beach Harbor with no landfill. The numerical model calculates harbor oscillation for harbors of arbitrary shape and variable depth. A finite element grid which covered the immediate vicinity of the breakwater-protected tanker terminal area was used to calculate the response of this area to incident waves with periods from 30 sec to approximately 6 min.

DD FORM 1 JAN 73 **1473** EDITION OF 1 NOV 65 IS OBSOLETE

SECURITY CLASSIFICATION OF THIS PAGE *(When Data Entered)*

038100
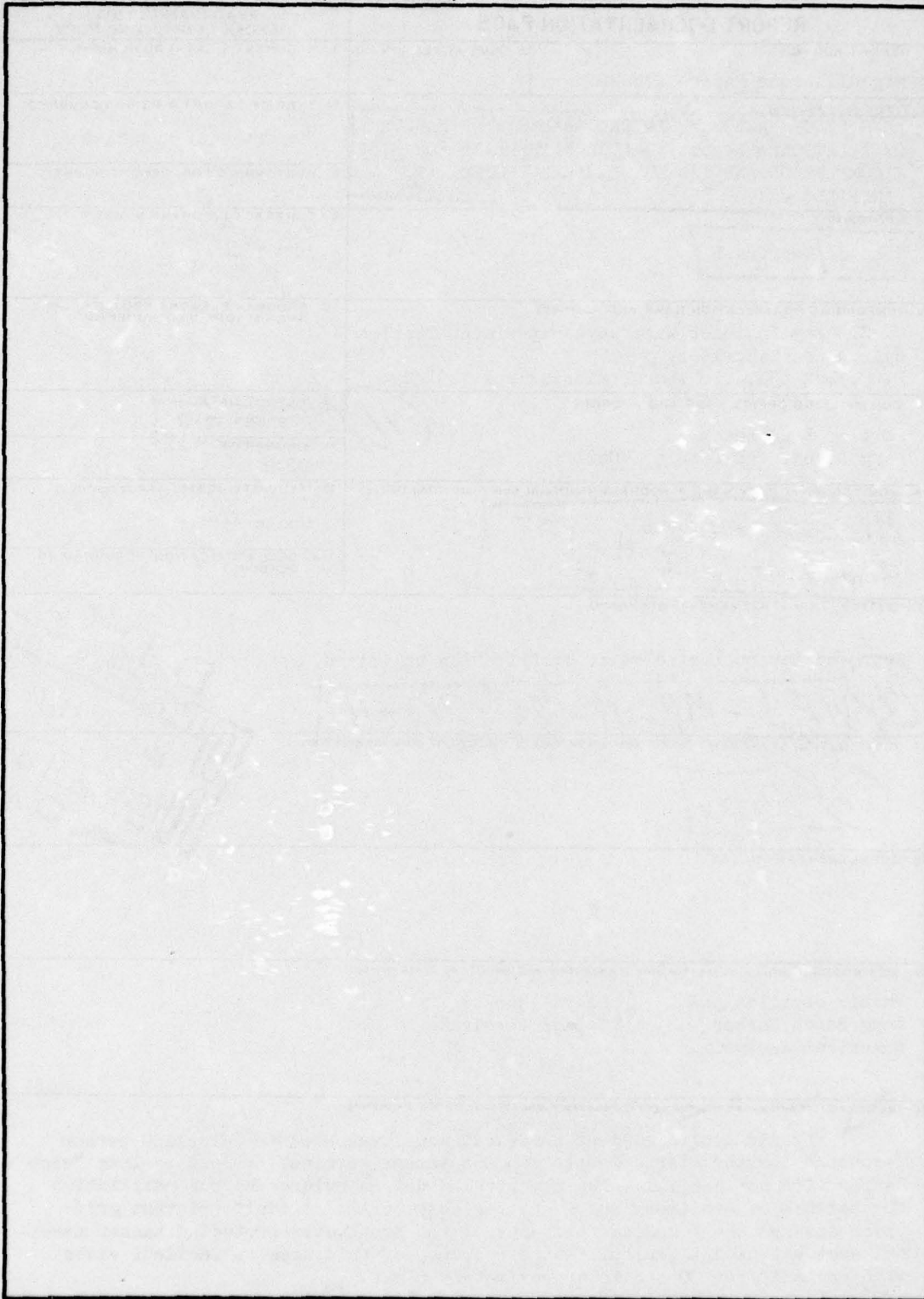
PREFACE

The investigation reported herein was authorized by the Long Beach Port Authority under a contract, Agreement No. WES 76-4, dated 12 September 1975.

The investigation was conducted from September 1975 to October 1976 by personnel of the Hydraulics Laboratory, U. S. Army Engineer Waterways Experiment Station (WES), under the direction of Mr. H. B. Simmons, Chief of the Hydraulics Laboratory, Dr. R. W. Whalin, Chief of the Wave Dynamics Division, and Mr. C. E. Chatham, Chief of the Harbor Wave Action Branch. Dr. J. J. Wanstrath, Research Oceanographer, conducted the investigation and prepared this report. Mr. R. R. Bottin, Jr., aided in the development of the finite element grids. Drs. H. S. Chen and C. C. Mei of the Massachusetts Institute of Technology provided documentation of the hybrid finite element computer program they developed and materials to aid in its utilization.

Directors of WES during the investigation and the preparation and publication of this report were COL G. H. Hilt, CE, and COL John L. Cannon, CE. Technical Director was Mr. F. R. Brown.

1

CONTENTS

CONVERSION FACTORS, U. S. CUSTOMARY TO METRIC (SI)
UNITS OF MEASUREMENT

U. S. customary units of measurement used in this report can be converted
to metric (SI) units as follows:

Multiply	By	To Obtain
feet	0.3048	metres
acres	4046.856	square metres
cubic yards	0.7645549	cubic metres
feet per second	0.3048	metres per second
square feet per second	0.09290304	square metres per second
feet per second per second	0.3048	metres per second per second

ALTERNATE PLANS FOR PIER J COMPLETION AND TANKER
TERMINAL PROJECT (NO LANDFILL)

PART I: INTRODUCTION

Objective

1.   The objective of this study was to investigate by use of a
numerical model[*] the response to long-period wave excitation of a pro-
posed modification to Pier J in Long Beach Harbor for oil tanker berth-
ing and general cargo facilities. Wave-height amplification factors ar
normalized maximum current velocities were plotted versus wave period a
the three proposed tanker terminals to ascertain whether or not signifi
cant harbor oscillations might occur that would pose problems to ship
berthing.

Scope

2.   The tanker terminal configuration under consideration in
this study (plan 3) omits the landfill of approximately 111 acres[**]
(11 million cu yd) on the end of Pier J (Figure 1). Three other plans
have been tested[†] and are summarized on the following page (Figures 2
and 3).

---

[*] H. S. Chen and C. C. Mei, "Oscillations and Wave Forces in an Off-
shore Harbor (Applications of the Hybrid Finite Element Method to
Water-Wave Scattering)," Report No. 190, 1974, Massachusetts Institu
of Technology, Cambridge, Mass.

[**] A table of factors for converting U. S. customary units of measure-
ment to metric (SI) units is presented on page 3.

[†] J. R. Houston, "Long Beach Harbor Numerical Analysis of Harbor
Oscillations; Alternate Plans for Pier J Completion and Tanker Termi
nal Project," Miscellaneous Paper H-76-20, Report 2, Sep 1976,
U. S. Army Engineer Waterways Experiment Station, CE, Vicksburg,
Miss.

Plan	Designation	Description
1A	Scheme STFP 2	300-ft opening (1000-ft-long east breakwater) at the east end of the 62-ft-deep basin requiring 111 acres of landfill on Pier J.
1B	Scheme STFP 2	600-ft opening (700-ft-long east breakwater) at east end of the 62-ft-deep basin requiring 111 acres of landfill on Pier J.
2	Scheme STFP 3	1100-ft opening (1300-ft-long east breakwater) at the east end of the 62-ft-deep basin requiring 50 acres of landfill on Pier J.
3		1700-ft opening (700-ft-long east breakwater) at the east end of the 62-ft-deep basin requiring no landfill on Pier J.

3. For all plans, including plan 3, the incident waves were directed normal to the main breakwater with periods from 30 sec to 300-360 sec. For this study, wave amplitudes and currents were calculated every 6 sec for the period range. Resonant peaks also were defined by considering incident wave periods in increments as low as 1 sec. Frequency response curves are given comparing plans 1B and 3 in detail, and a tabular summary of resonant peaks is presented for all plans.
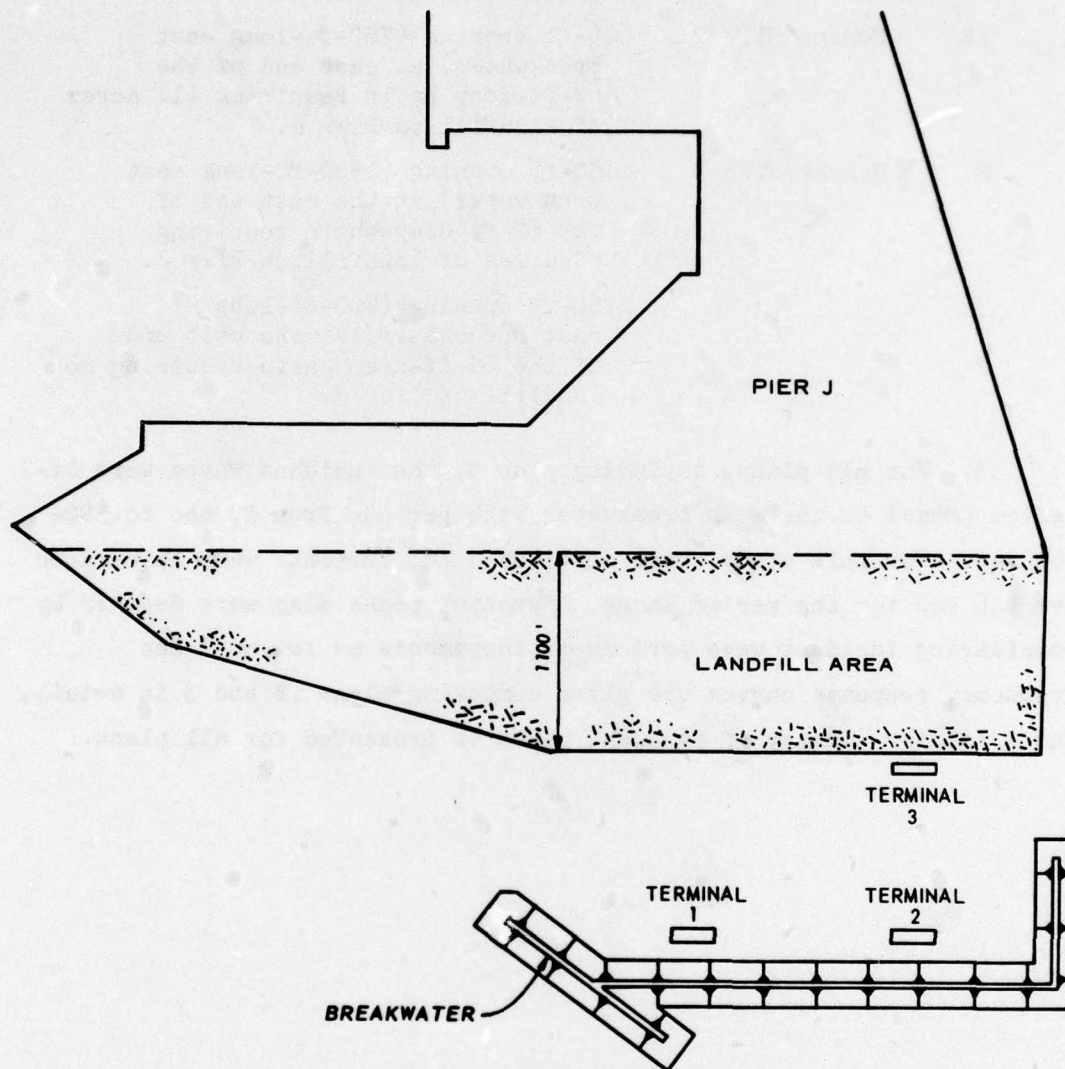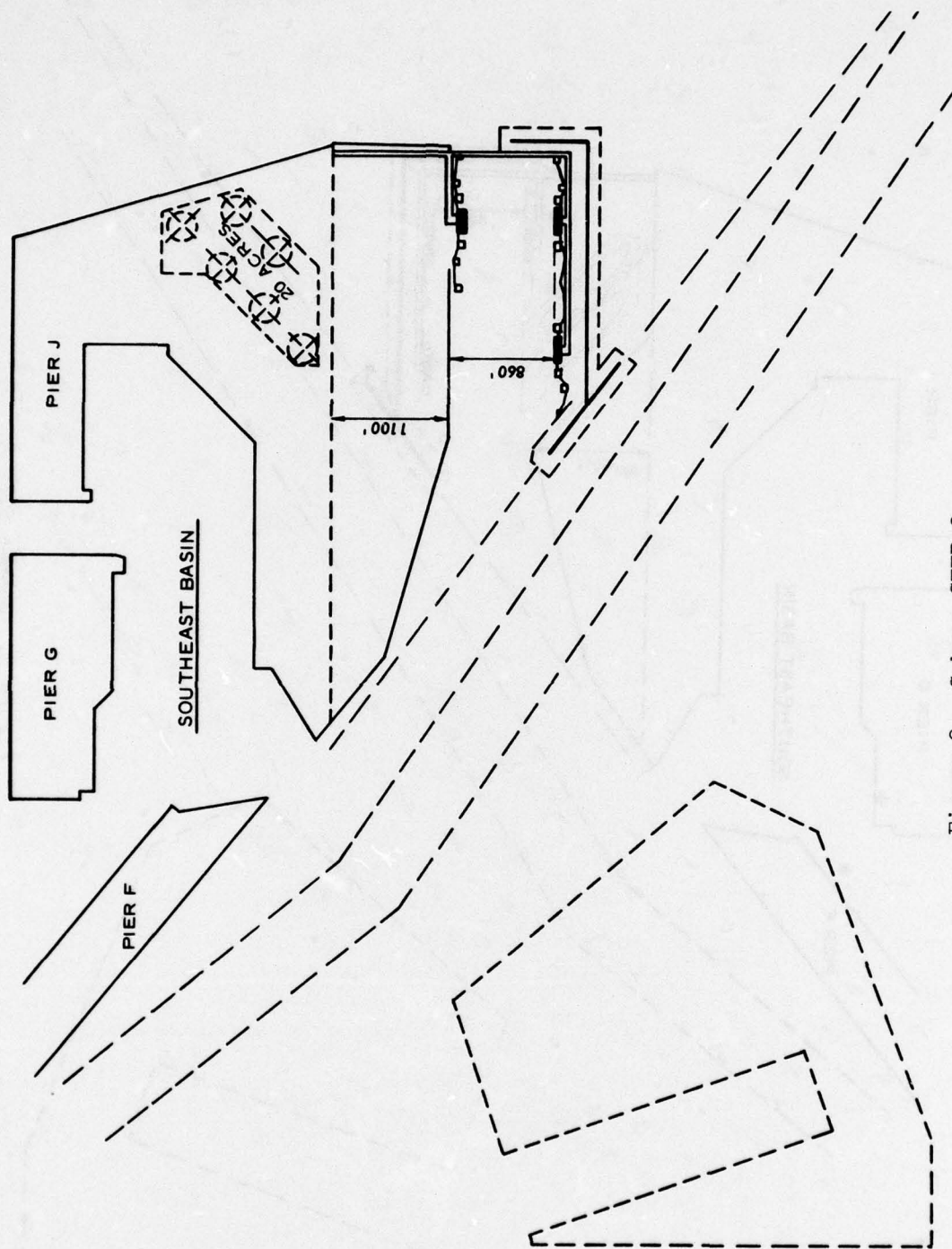
Figure 1. Location of tanker terminals
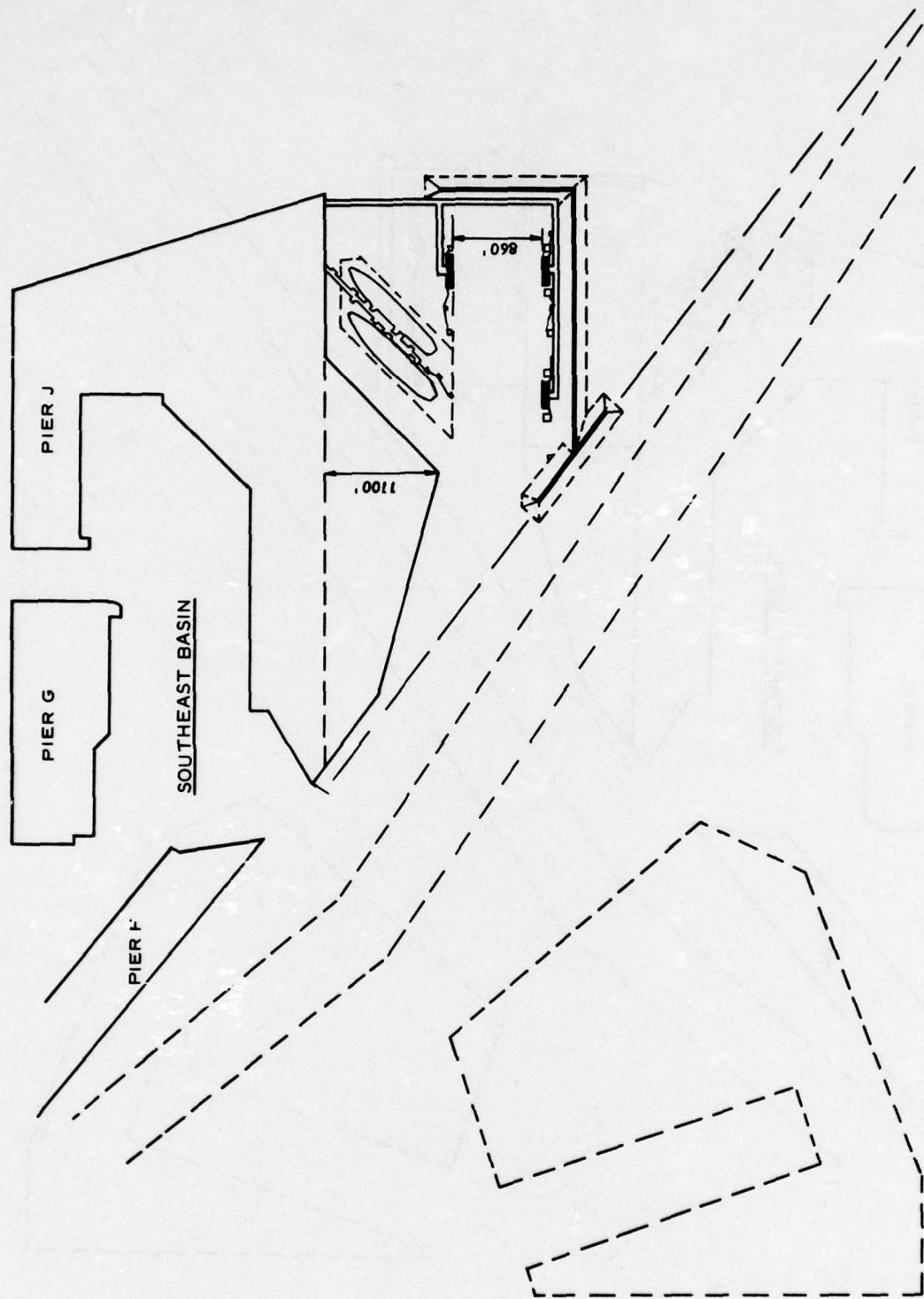
Figure 2. Scheme STFP 2

7

Figure 3. Scheme STFP 3

PIER J

PIER G

PIER F

SOUTHEAST BASIN

1100'

098'

8

PART II: NUMERICAL MODEL

4.   The response of the tanker terminal basin to long wave excita-
tion was determined by using a hybrid finite element numerical model
developed recently by Chen and Mei.* The model solves the following
generalized Helmholtz equation:

$$\nabla \cdot \left[h(x,y)\nabla\phi(x,y)\right] + \frac{w^2}{g} \phi(x,y) = 0 \tag{1}$$

where $\phi(x,y)$** is the velocity potential defined by $u(x,y) = -\nabla\phi(x,y)$,
with $u(x,y)$ being a two-dimensional velocity vector and w an angular
frequency. Equation 1 governs small amplitude undamped oscillations
of water in a basin of arbitrary shape and variable depth forced by
periodic long waves. It has been further assumed that the flow is
irrotational.

5.   The boundary condition along the shoreline and along the de-
tached breakwater surrounding the three proposed berths is that the
normal component of the velocity be equal to zero. Therefore, the
breakwater is considered as a solid barrier. No special boundary con-
ditions are made to account for any vessels utilizing the tanker termi-
nal basin, since for long-period waves of small amplitude ship heave
equals incident wave height. Therefore, the vessels do not signifi-
cantly alter the oscillation characteristics of the basin.

6.   The Helmholtz equation:

$$\nabla^2\phi(x,y) + \frac{w^2}{gh} \phi(x,y) = 0 \tag{2}$$

is the governing equation for a constant-depth ocean region outside the
basin.

7.   For a harbor in a semi-infinite ocean with a straight coastline

---

*   Chen and Mei, op. cit.
** For convenience, symbols and unusual abbreviations are listed and
    defined in the Notation (Appendix A).

9

there is an incident, reflected, and scattered wave. The scattered wave has a velocity potential ϕ_s given by

$$\phi_s = \sum_{n=0}^{\infty} \alpha_n H_n(kr) \cos n\theta \tag{3}$$

where α_n are unknown coefficients and $H_n(kr)$ are Hankel functions of the first kind of order n. ϕ_s satisfies the radiation condition that the scattered wave must behave as an outgoing wave at infinity. This condition is known as the Sommerfeld radiation condition and may be expressed mathematically as follows:

$$\lim_{r \to \infty} \sqrt{r} \left(\frac{\partial}{\partial r} - ik \right) \phi_s = 0 \tag{4}$$

8. Chen and Mei used a calculus of variations approach and obtained a Euler-Lagrange formulation of the boundary value problem. The following functional with the property that it is stationary with respect to arbitrary first variations of $\phi(x,y)$ was constructed by Chen and Mei:

$$F(\phi) = \iint \frac{1}{2} \left\{ h(\nabla \phi)^2 - \frac{w^2}{g} \phi^2 \right\} dA$$

$$+ \frac{1}{2} \oint \left\{ h(\phi_R - \phi_I) \frac{\partial(\phi_R - \phi_I)\}}{\partial n_a} \right. da - \oint \left\{ h\phi_a \frac{\partial(\phi_R - \phi_I)\}}{\partial n_a} \right. da$$

$$- \oint \left\{ h\phi_a \frac{\partial \phi_I}{\partial n_a} \right\} da + \oint \left\{ h\phi_I \frac{\partial(\phi_R - \phi_I)\}}{\partial n_a} \right. da \tag{5}$$

where

    A = region inside the harbor

    $\oint$ = line integral

    ϕ_R = far field velocity potential

ϕ_I = velocity potential of the incident wave

n_a = unit normal vector outward from region A

a = boundary of region A

ϕ_a = total velocity potential evaluated on boundary a

Proof was given by Chen and Mei that the stationarity of this functional is equivalent to the original boundary value problem.

9. The integral equation obtained from extremizing the functional is solved by utilizing the finite element method. This method is a technique of numerical approximation which involves dividing a domain into a number of nonoverlapping subdomains which are called elements.

10. The solution of the problem is approximated within each element by suitable interpolation functions in terms of a finite number of unknown parameters. These unknown parameters are the values of the field variable $\phi(x,y)$ at a finite number of points which are called nodes. The relations for individual elements are combined into a system of equations for all unknown parameters.

11. In the region outside the basin, the velocity potentials are solved analytically in terms of unknown coefficients. The region is considered a single element with an "interpolation function" given by Equation 3. The infinite series is terminated at some finite value such that the addition of further terms does not significantly influence the calculated values of $\phi(x,y)$. The resulting equation is combined with the system of equations for unknown parameters at nodal points within the basin and this complete system is solved using Gaussian elimination matrix methods. $\eta(x,y)$ is related to $\phi(x,y)$ through the linearized dynamic free surface boundary condition

$$\eta(x,y) = -\frac{1}{g}\frac{\partial\phi(x,y)}{\partial t} \tag{6}$$

The horizontal velocity components have the following form:

$$u(x,y) = -\frac{g}{w}\frac{\partial\eta(x,y)}{\partial x}; \quad v(x,y) = -\frac{g}{w}\frac{\partial\eta(x,y)}{\partial y} \tag{7}$$

11

12.   The hybrid finite element method (so named by Chen and Mei
because the method involves the combination of analytical and finite
element numerical solutions) is a steady-state solution of the boundary
value problem. The response of a harbor to an arbitrary forcing func-
tion can be easily determined within the framework of a linearized
theory.

13. Locations of the three tanker terminals are shown in Figure 1. The finite element grid for plan 3 is presented in Plate 1. Plans 1B and 3 are similar regarding the dimensions of the breakwater, but relative to the east harbor opening and basin dimensions, plan 2 is more similar to plan 3 than is plan 1B to plan 3.

14. The frequency response curves of the normalized maximum current velocity (NMCV) versus incident wave period at each tanker terminal for plans 1B and 3 are shown in Plates 2-4. The current velocity multiplied by the incident wave amplitude in feet gives velocity in units of feet per second. The velocities have no vertical component or variation since only long waves are considered. Also, the maximum velocity over one wave period is plotted. Plates 5-7 are the frequency response curves of the wave-height amplification factor (WHAF) at each tanker terminal. WHAF is defined at a point inside the basin as the wave height at the point divided by twice the incident wave height. This definition of amplification factor is traditional and is a result of the fact that the standing wave height for a straight coast with no harbor would be twice the incident wave due to the superposition of the incident and reflected waves.

15. The following tabulation gives peak values of NMCV and WHAF and the resonant wave period for the various Pier J configurations. "Blanks" in the tabulation reflect either the absence of a resonant condition or that NMCV or WHAF did not exceed 1.0. Several observations regarding either the NMCV or WHAF are apparent. For the wave period range under consideration, plan 3 exhibits four resonant wave periods (44, 66, 124, and, nominally, 340 sec) at each tanker terminal. If a resonant condition is observed at a particular terminal in either plan 1A, 1B or 2, it is also observed in plan 3 at a slightly different resonant wave period. If the peak is absent in plan 3, it is also absent in the other plans. The peaks in plan 3 are generally greater than those in the other plans for the first three resonant periods. However, for the last resonant period (that covering a wave period range

13

from approximately 2 min to well over 6 min, and where a greater pro-
portion of wave energy is concentrated per wave period interval)
plan 3 always exhibits a lower peak value than those in the other plans.

STFP 2 (300) Plan 1A		STFP 2 (600) Plan 1B		STFP 3 Plan 2		Plan 3	
NMCV	WHAF	NMCV	WHAF	NMCV	WHAF	NMCV	WHAF
Terminal 1							
--	--	--	--	1.20 (36)	--	3.22 (44)	2.08 (44)
--	--	--	--	2.26 (56)	--	3.47 (66)	3.25 (66)
--	--	--	--	1.52 (106)	1.00 (102)	2.01 (124)	2.93 (124)
2.45 (280)	3.93 (285)	1.93 (260)	3.22 (268)	2.50 (350)	2.75 (349)	1.55 (330)	2.22 (348)
Terminal 2							
--	--	--	--	--	--	2.72 (44)	1.78 (44)
--	--	1.65 (59)	--	4.36 (56)	1.61 (56)	3.43 (66)	2.39 (66)
--	--	--	--	2.55 (106)	2.55 (106)	1.94 (124)	3.40 (124)
--	3.49 (286)	--	3.66 (264)	--	3.24 (352)	--	2.33 (348)
Terminal 3							
--	--	--	--	1.24 (46)	--	3.68 (44)	--
--	--	1.00 (58)	1.12 (58)	1.90 (57)	3.72 (54)	2.79 (66)	3.80 (66)
--	--	--	--	3.48 (108)	--	4.80 (125)	--
1.78 (280)	3.69 (284)	1.74 (260)	3.32 (270)	--	3.12 (360)	1.72 (330)	2.25 (348)

Note: Numbers in parentheses are the wave period in seconds at the peak.

16. Plan 3 evidently establishes a harbor of sufficient length,
width, and depth that the first four principal harmonics can be nicely
generated. These harmonics are portrayed in the contours of the WHAF
shown in Plates 8-11. Three nodal areas oriented parallel to the long
axis of the basin are generated for the first resonant wave period
(44 sec). The binodal and uninodal conditions (each nodal area oriented
east-west as in the trinodal oscillation) occur for the next two reso-
nant periods (66 and 124 sec). Antinodes form along the inside of the
detached breakwater (the corners of which do not particularly accentuate
the oscillation) and along the face of Pier J. However, for the 6-min
resonant period, a single antinode is formed in the middle of the basin,
oriented north-south. This is the only harmonic which is similarly
well developed and oriented in all the plans.

17. Plates 12-15 present the NMCV patterns for the resonant peaks
of plan 3. Comparison of these plates with those from the previous
study* for plans 1A, 1B, and 2 shows the currents are generally larger

---

*   Houston, op. cit.

14

in the basin interior (especially along nodal lines) for the first three resonant oscillations (44, 66, and 124 sec) of plan 3 than for the other plans. However, the current patterns for plan 3 exhibit no particular modification at the interior breakwater corners or high velocities at the ends of the breakwater as observed in the other plans. For the 6-min resonant condition of plan 3, the current pattern is similar to that of the other plans except that as observed from the preceding tabulation, the magnitudes of the current velocities at the tanker terminals are smaller than those in any other plan.

18. In summary, larger resonant oscillations occur in plan 3 than in the other plans for wave periods less than 2 min. The basin is of sufficient length, width, and depth that three resonant periods (44, 66, and 124 sec) can be excited. The other plans are of such basin dimensions that one or more of the principal harmonics are not excited or are excited to a lesser degree than that in plan 3. Plan 3, however, excites the first principal east-west oscillation (6 min) with lower wave amplifications and current velocities at the tanker terminals than those of any other plan.
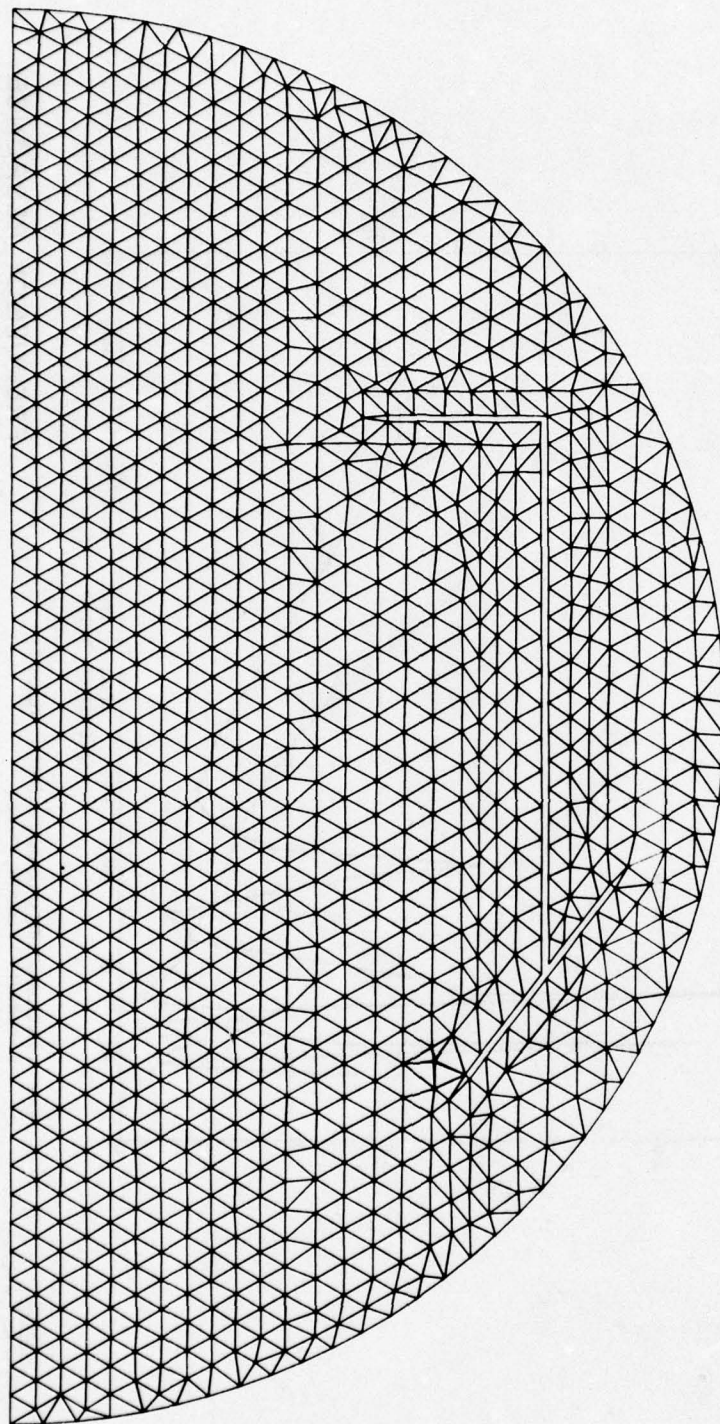
19. The resonant peaks of the normalized maximum current velocity and wave-height amplification factor do not appear to preclude safe berthing at the terminal areas, given proper mooring practice and adequate mooring equipment. The resonant response peaks for incident waves with periods less than 2 min are generally larger for plan 3 than for the other plans. Plan 3, however, exhibits smaller response peaks for the longer period (6 min) oscillation than in the other plans. Whether or not incident waves in this period range excite significant ship motion depends upon ship characteristics (length, width, and draft) and mooring conditions (type of lines, amount of slack, etc.).

20. The incident wave spectrum and ship response versus wave period curves must be known in addition to the harbor response curves of Plates 2-7 to definitively determine whether or not ship surge problems resulting from harbor resonance will occur in the tanker terminal basin.

21. Until detailed information on moored ship response as a function of incident wave amplitude and frequency is obtained, definitive conclusions on the precise amount of ship motion cannot be made. However, the oscillations of the tanker terminal basin are significantly smaller than the typical oscillation noted throughout the Los Angeles and Long Beach Harbors complex in the numerical study performed for the entire harbor complex.* Thus, it is reasonable to assume, prior to obtaining additional data on moored ship response, that satisfactory berthing conditions will obtain in the tanker terminal basin.

---

* J. R. Houston, "Long Beach Harbor Numerical Analysis of Harbor Oscillations; Existing Conditions and Proposed Improvements," Miscellaneous Paper H-76-20, Report 1, Sep 1976, U. S. Army Engineer Waterways Experiment Station, CE, Vicksburg, Miss.

FINITE ELEMENT GRID
NO LANDFILL
PLAN 3

SCALE IN FEET

NOTE NUMBER OF NODE POINTS = 1123
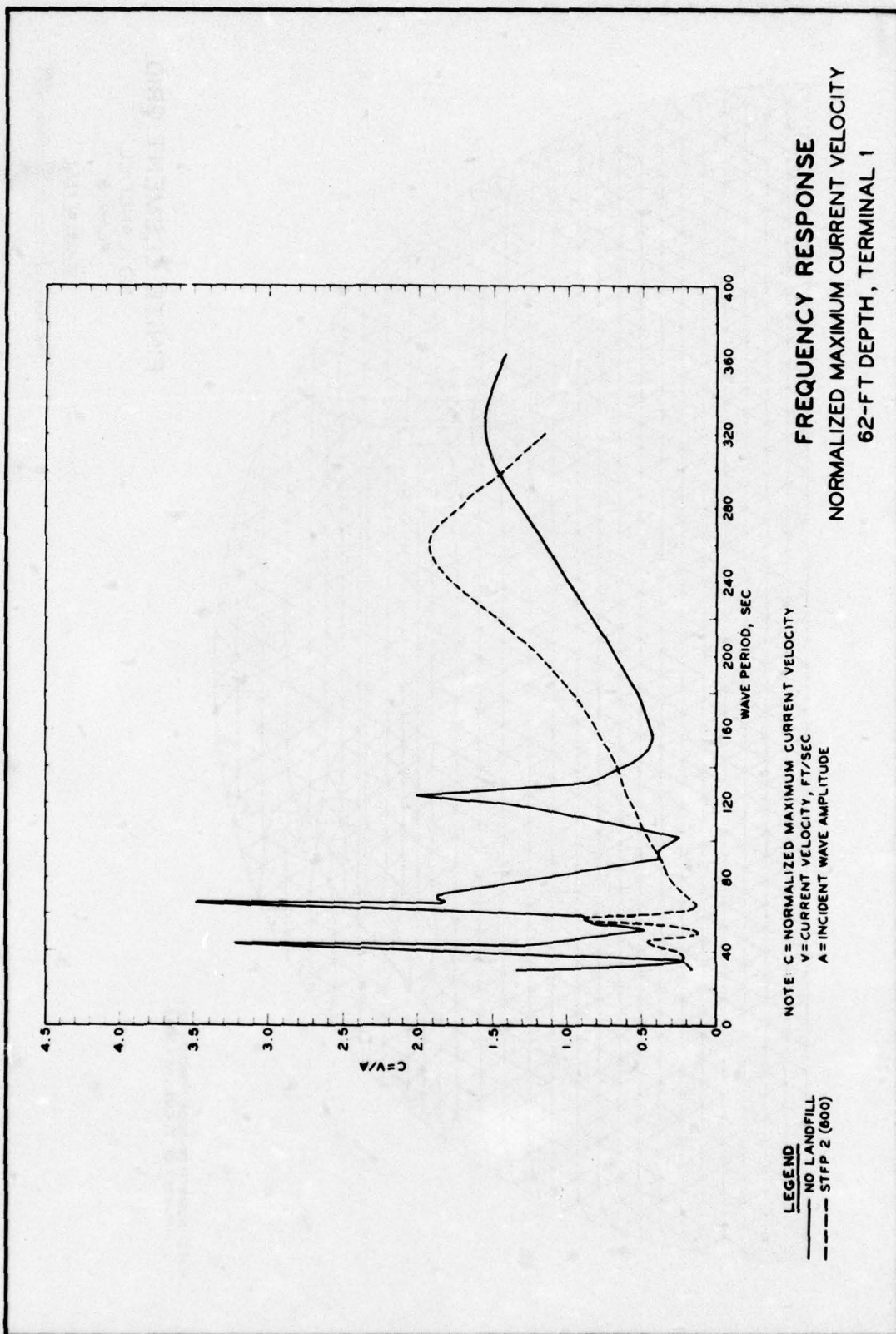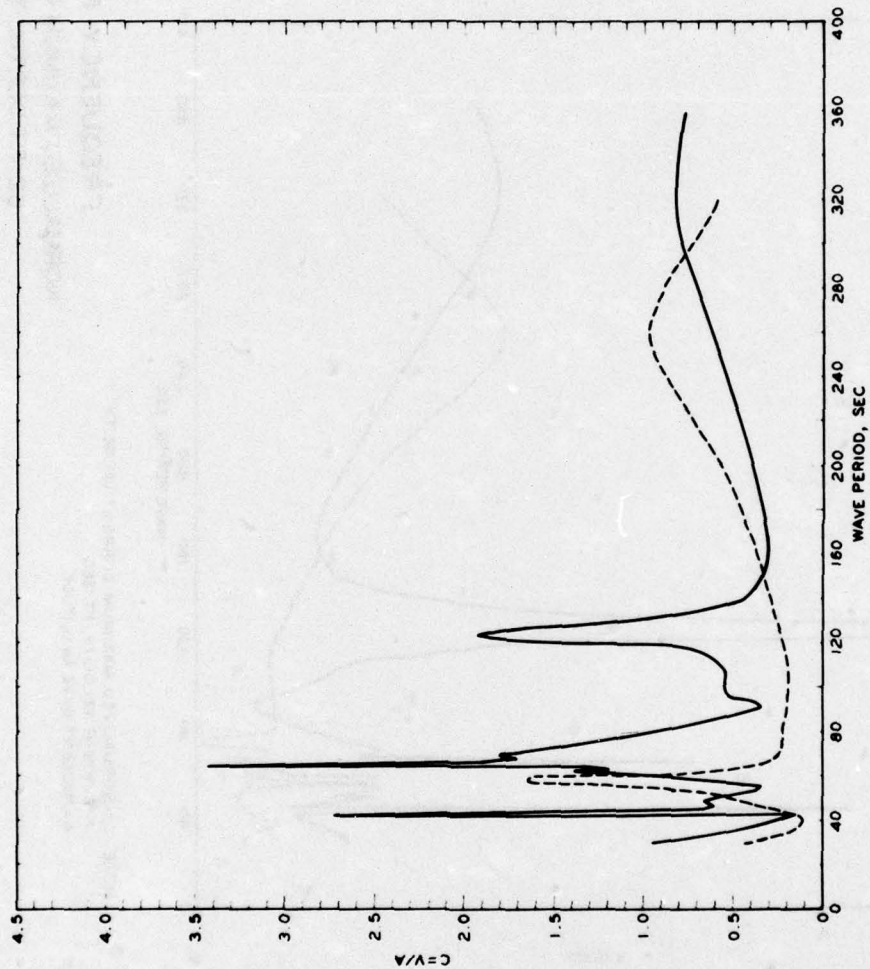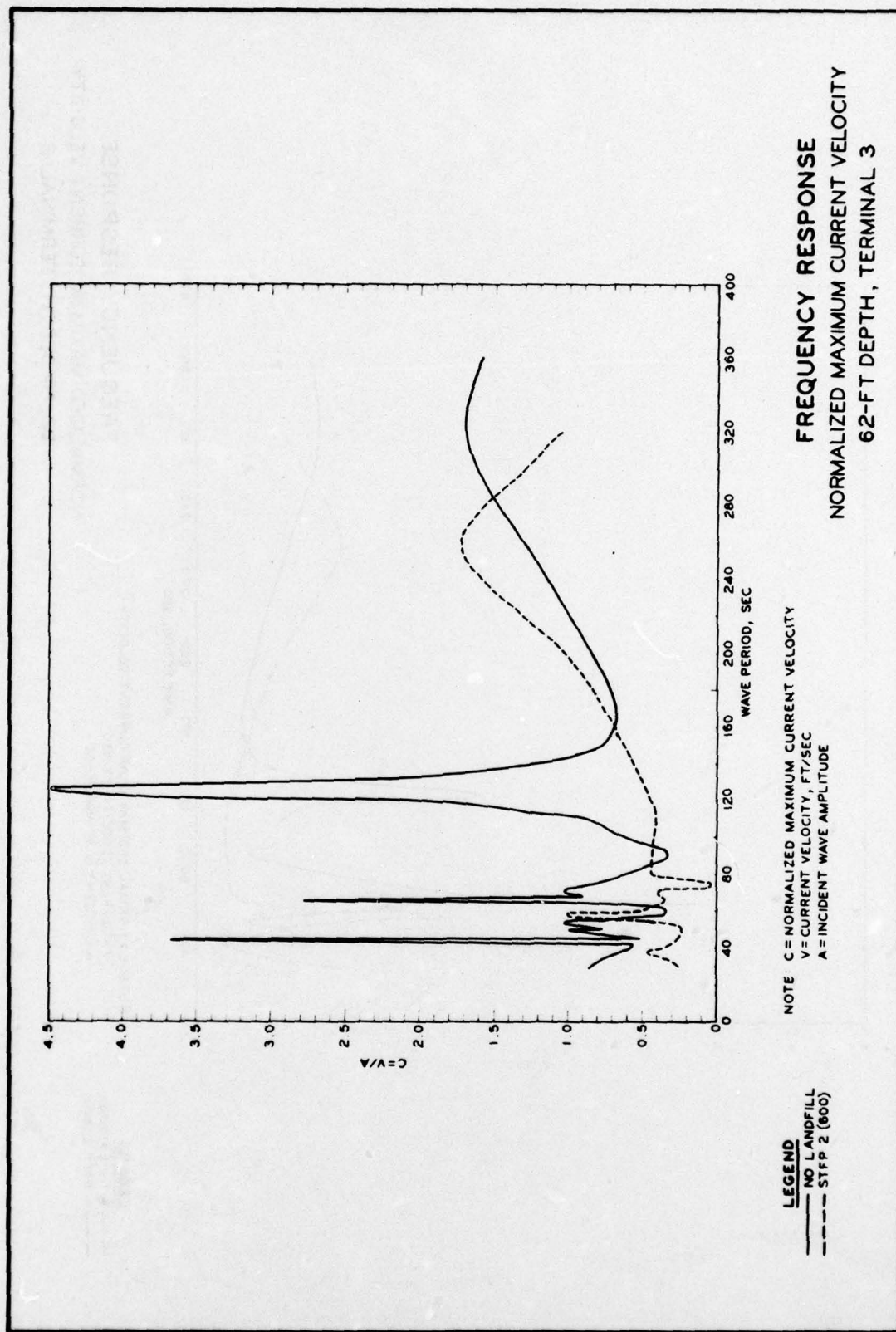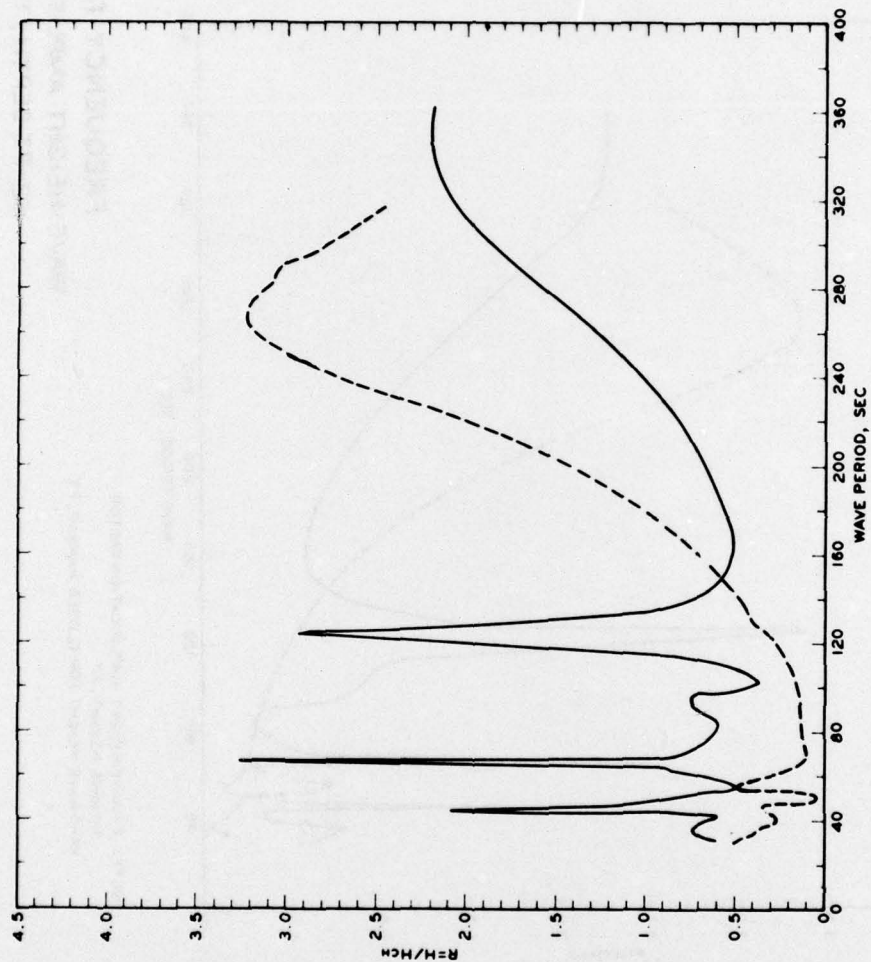NUMBER OF ELEMENTS = 2063

PLATE 1

FREQUENCY RESPONSE
NORMALIZED MAXIMUM CURRENT VELOCITY
62-FT DEPTH, TERMINAL 1

NOTE: C = NORMALIZED MAXIMUM CURRENT VELOCITY
V = CURRENT VELOCITY, FT/SEC
A = INCIDENT WAVE AMPLITUDE

LEGEND
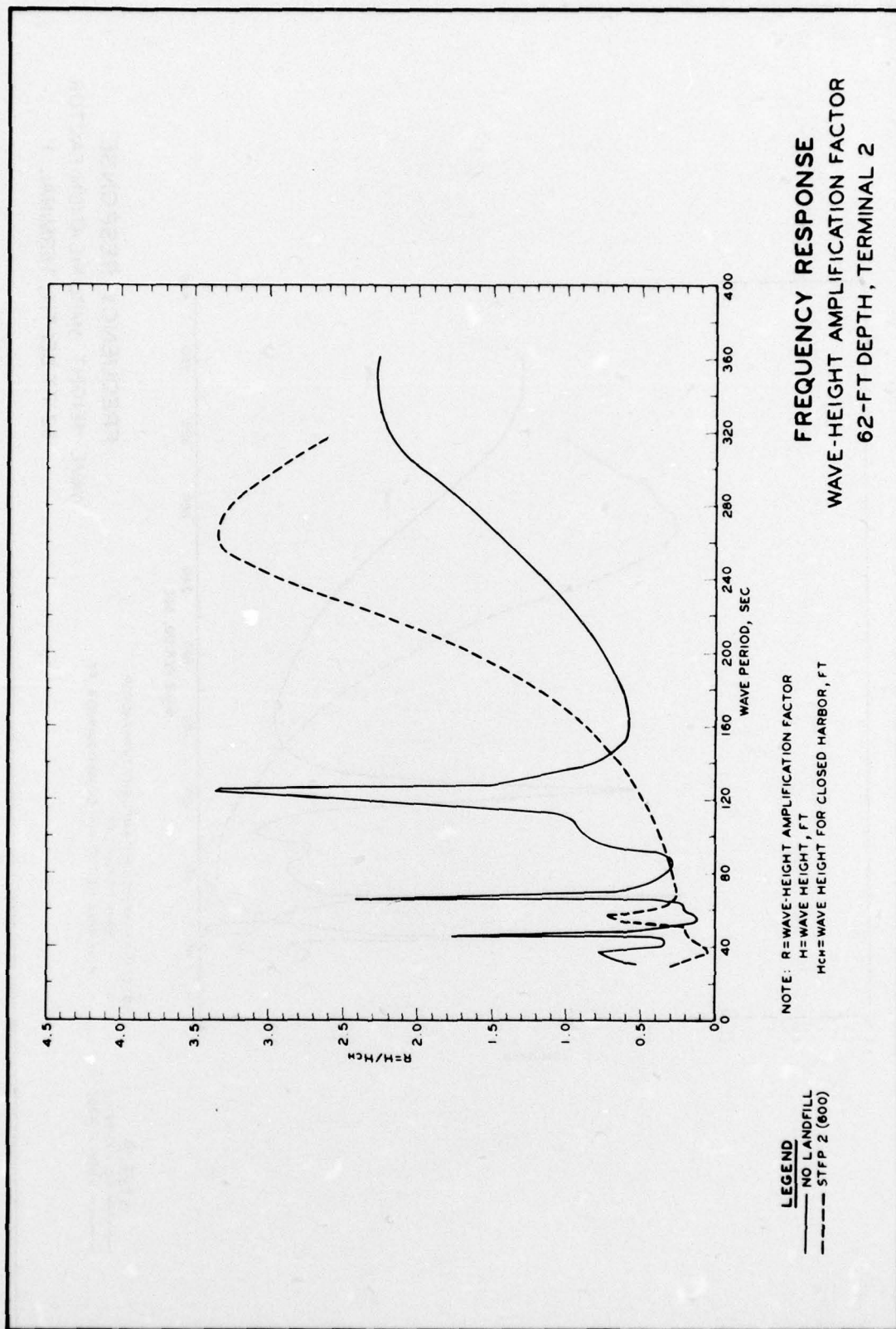————— NO LANDFILL
— — — STFP 2 (800)

PLATE 2

FREQUENCY RESPONSE

NORMALIZED MAXIMUM CURRENT VELOCITY

62-FT DEPTH, TERMINAL 2

WAVE PERIOD, SEC

C = V/A

NOTE: C = NORMALIZED MAXIMUM CURRENT VELOCITY
V = CURRENT VELOCITY, FT/SEC
A = INCIDENT WAVE AMPLITUDE

LEGEND
—— NO LANDFILL
--- STFP 2 (600)

PLATE 3

FREQUENCY RESPONSE
NORMALIZED MAXIMUM CURRENT VELOCITY
62-FT DEPTH, TERMINAL 3

NOTE C = NORMALIZED MAXIMUM CURRENT VELOCITY
     V = CURRENT VELOCITY, FT/SEC
     A = INCIDENT WAVE AMPLITUDE

LEGEND
——— NO LANDFILL
– – – STFP 2 (800)

PLATE 4

FREQUENCY RESPONSE
WAVE-HEIGHT AMPLIFICATION FACTOR
62-FT DEPTH, TERMINAL 1

NOTE: R=WAVE-HEIGHT AMPLIFICATION FACTOR
H=WAVE HEIGHT, FT
Hcн=WAVE HEIGHT FOR CLOSED HARBOR, FT

LEGEND
——— NO LANDFILL
- - - - STFP 2 (800)

PLATE 5

FREQUENCY RESPONSE
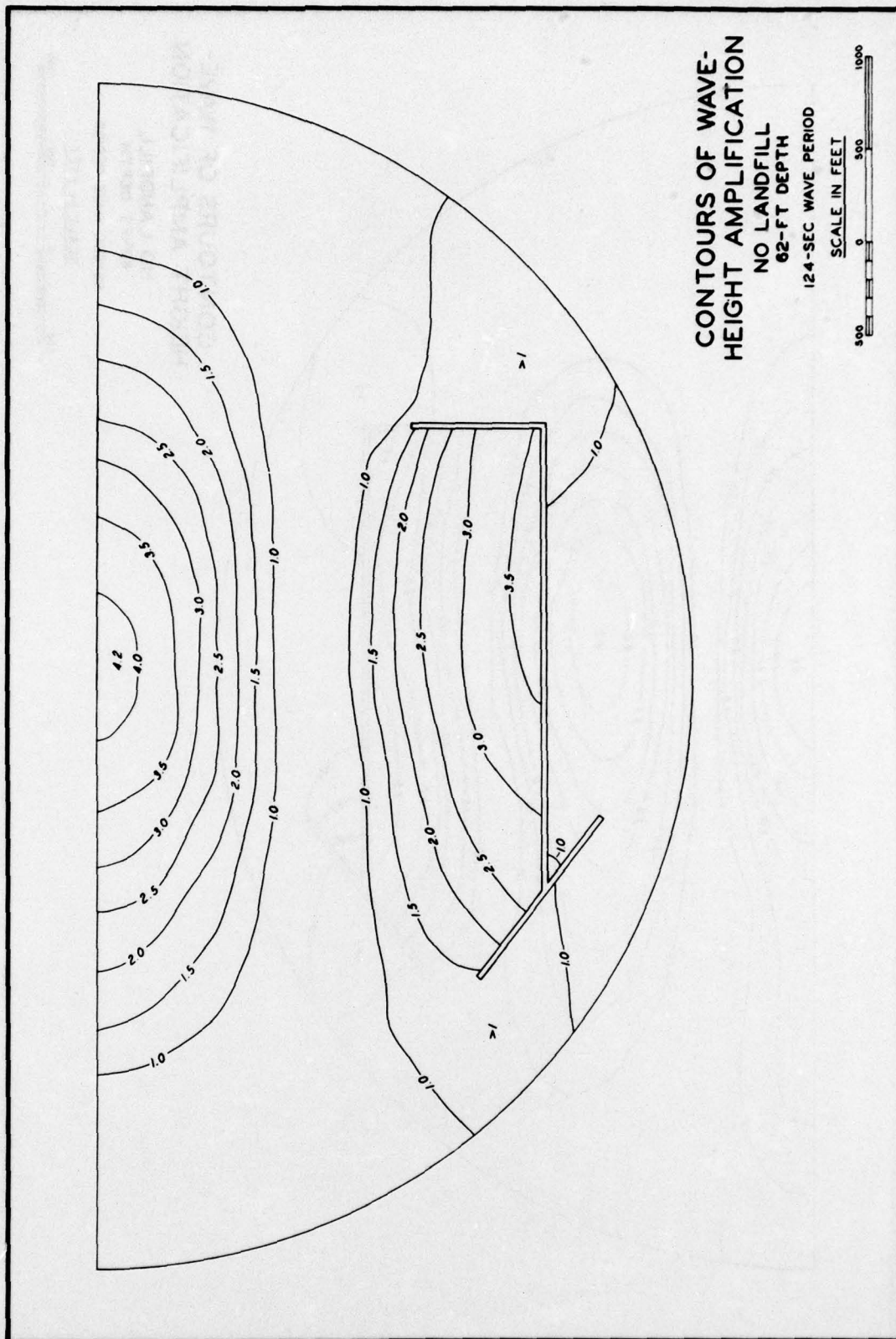WAVE-HEIGHT AMPLIFICATION FACTOR
62-FT DEPTH, TERMINAL 2

NOTE: R = WAVE-HEIGHT AMPLIFICATION FACTOR
H = WAVE HEIGHT, FT
Hcн = WAVE HEIGHT FOR CLOSED HARBOR, FT

LEGEND
——— NO LANDFILL
---- STFP 2 (800)

PLATE 6

FREQUENCY RESPONSE
WAVE-HEIGHT AMPLIFICATION FACTOR
62-FT DEPTH, TERMINAL 3

NOTE: R = WAVE-HEIGHT AMPLIFICATION FACTOR
H = WAVE HEIGHT, FT
H$_{CH}$ = WAVE HEIGHT FOR CLOSED HARBOR, FT

LEGEND
——— NO LANDFILL
— — — STEP 2 (600)

PLATE 7

CONTOURS OF WAVE-
HEIGHT AMPLIFICATION
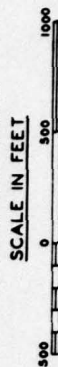
NO LANDFILL
62-FT DEPTH
44-SEC WAVE PERIOD

SCALE IN FEET

PLATE 8

CONTOURS OF WAVE-
HEIGHT AMPLIFICATION

NO LANDFILL
62-FT DEPTH

66-SEC WAVE PERIOD

SCALE IN FEET

PLATE 9

CONTOURS OF WAVE-
HEIGHT AMPLIFICATION
NO LANDFILL
62-FT DEPTH
124-SEC WAVE PERIOD

SCALE IN FEET
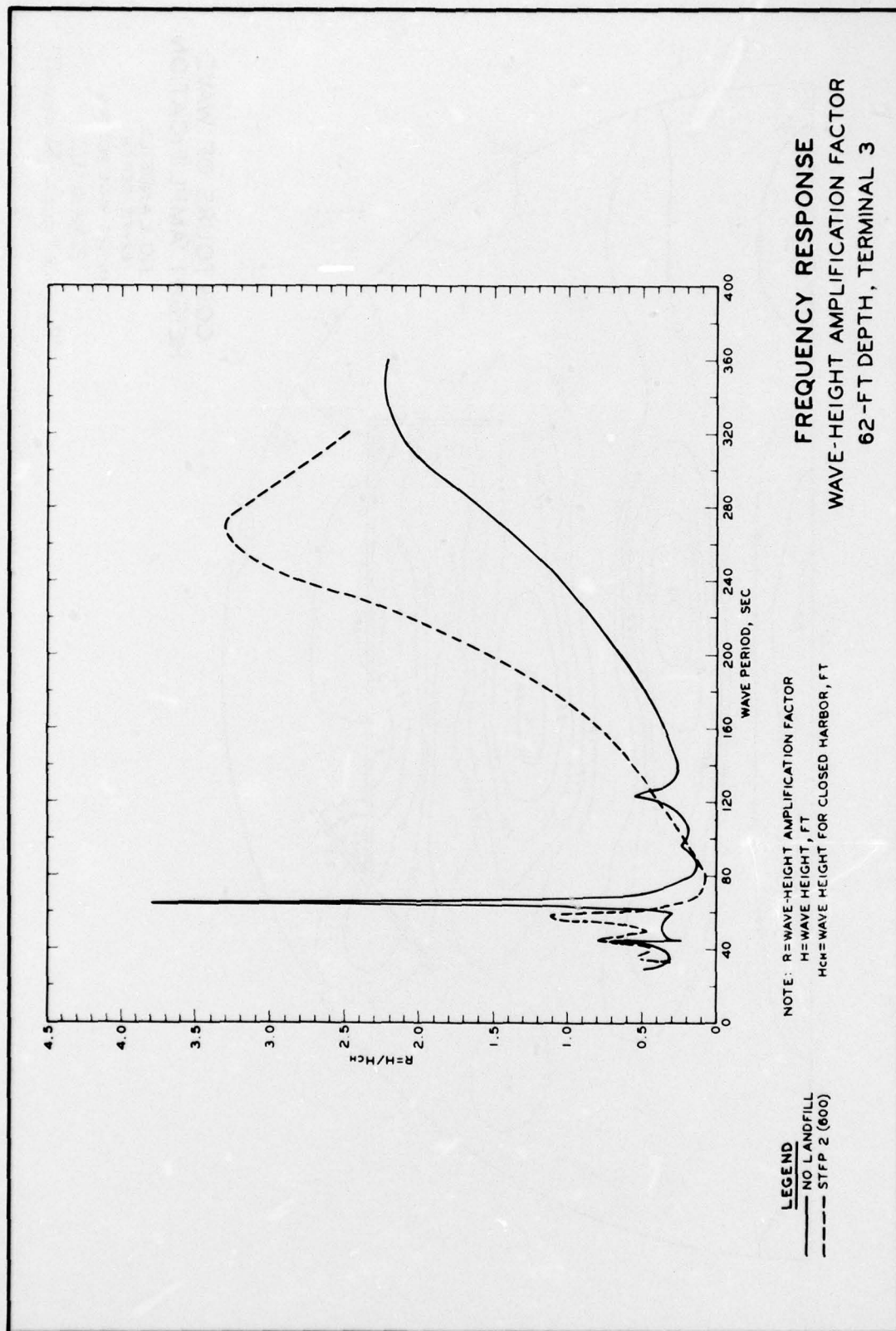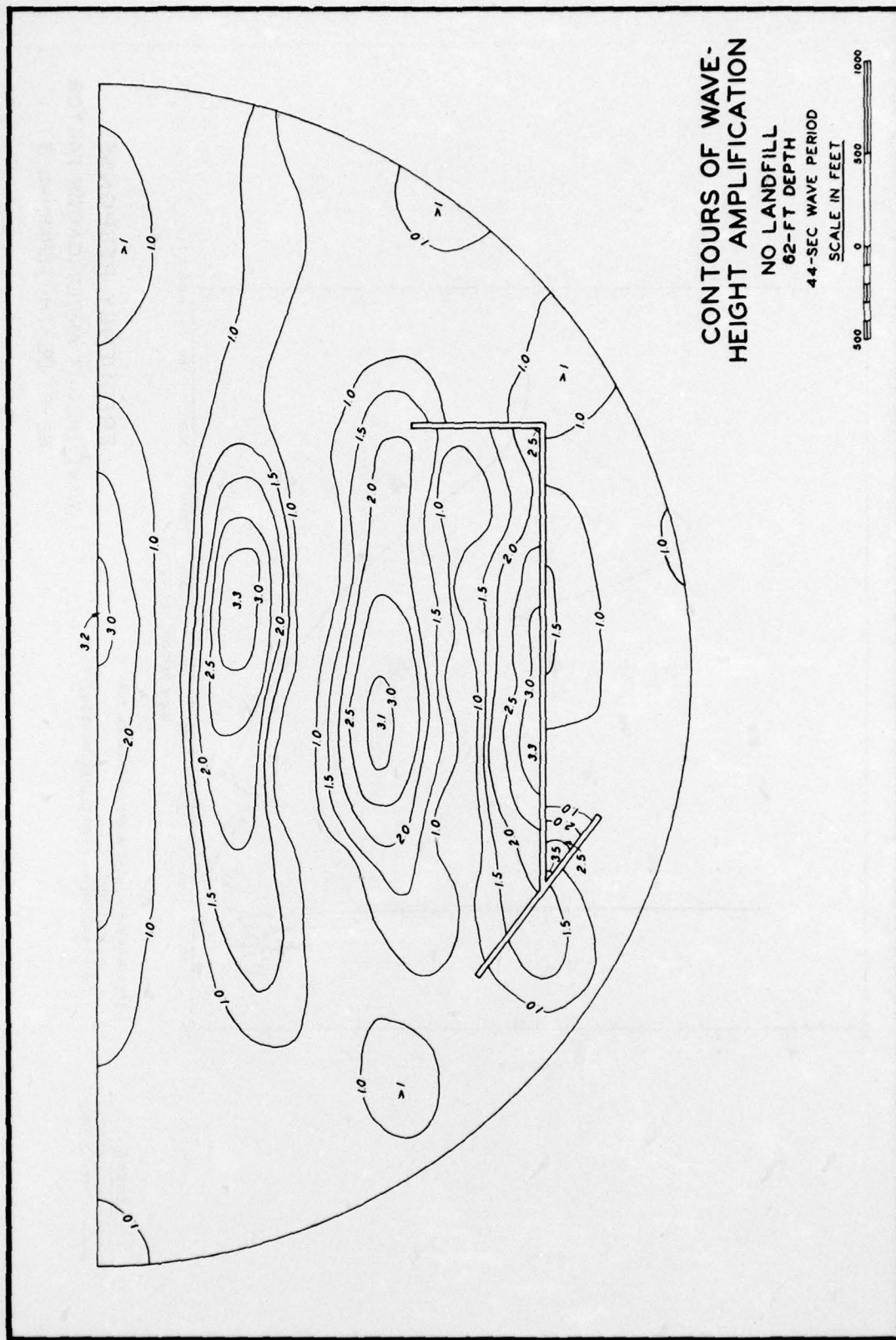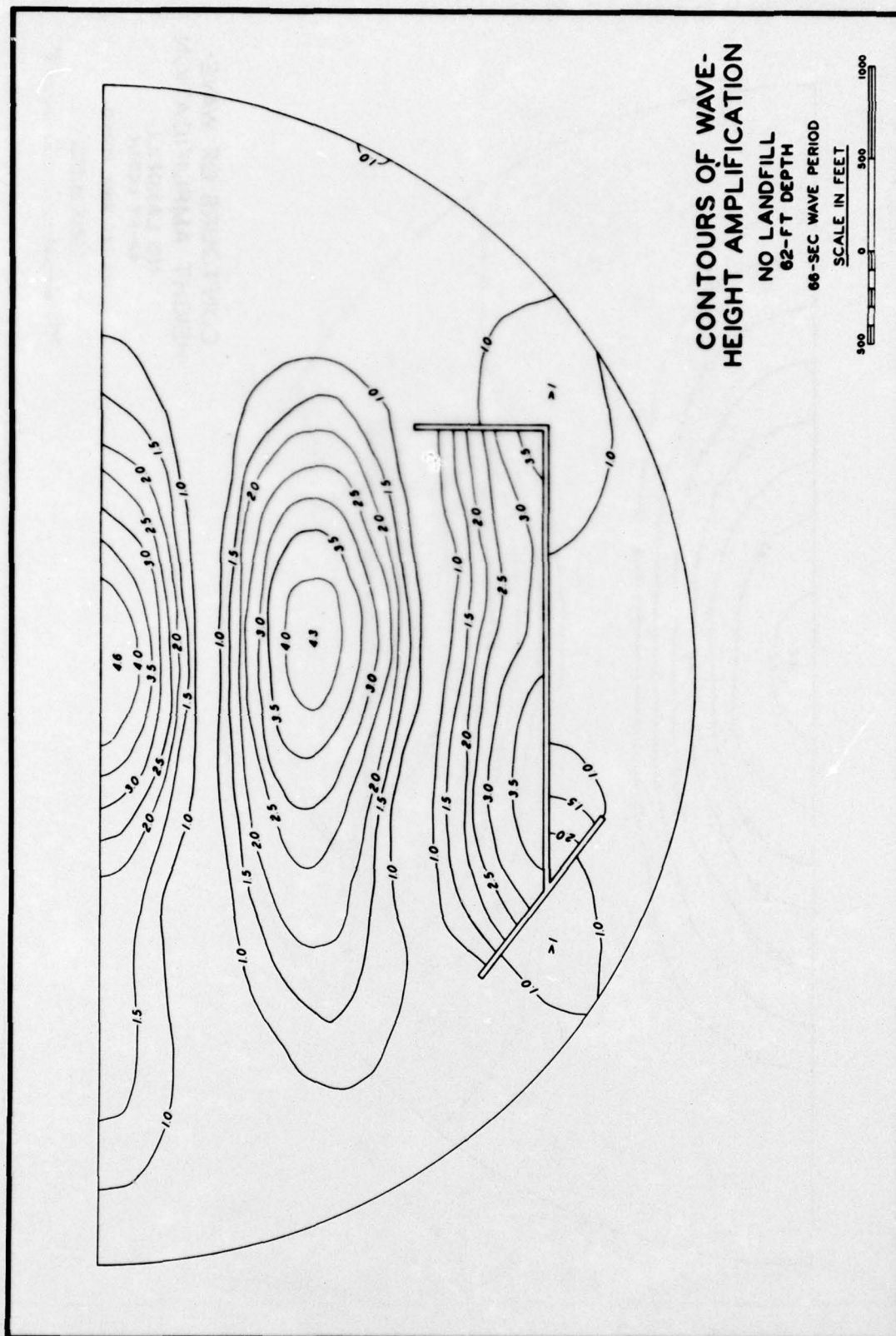
PLATE 10

CONTOURS OF WAVE-
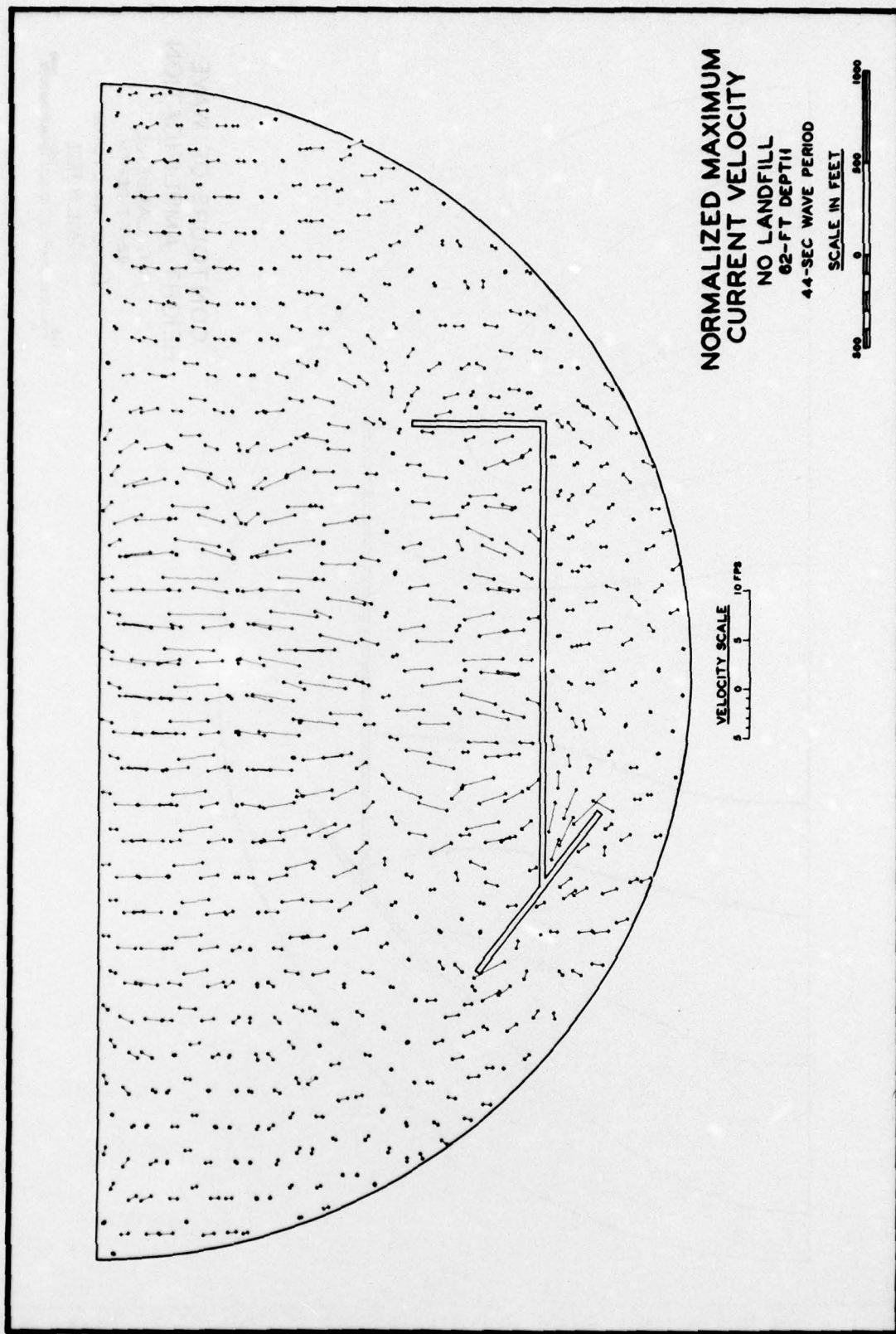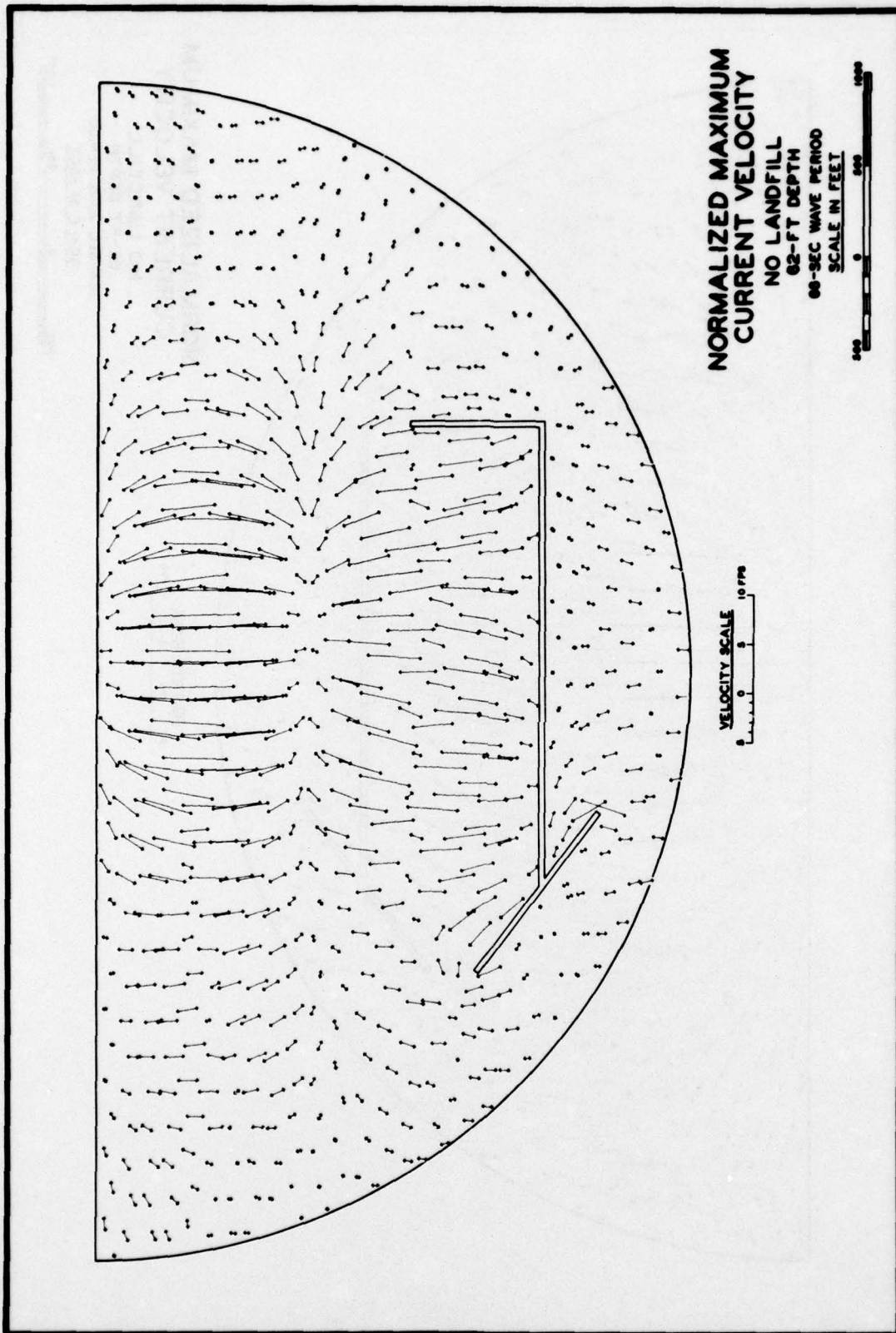HEIGHT AMPLIFICATION
NO LANDFILL
62-FT DEPTH
339-SEC WAVE PERIOD
SCALE IN FEET

PLATE 11

NORMALIZED MAXIMUM
CURRENT VELOCITY
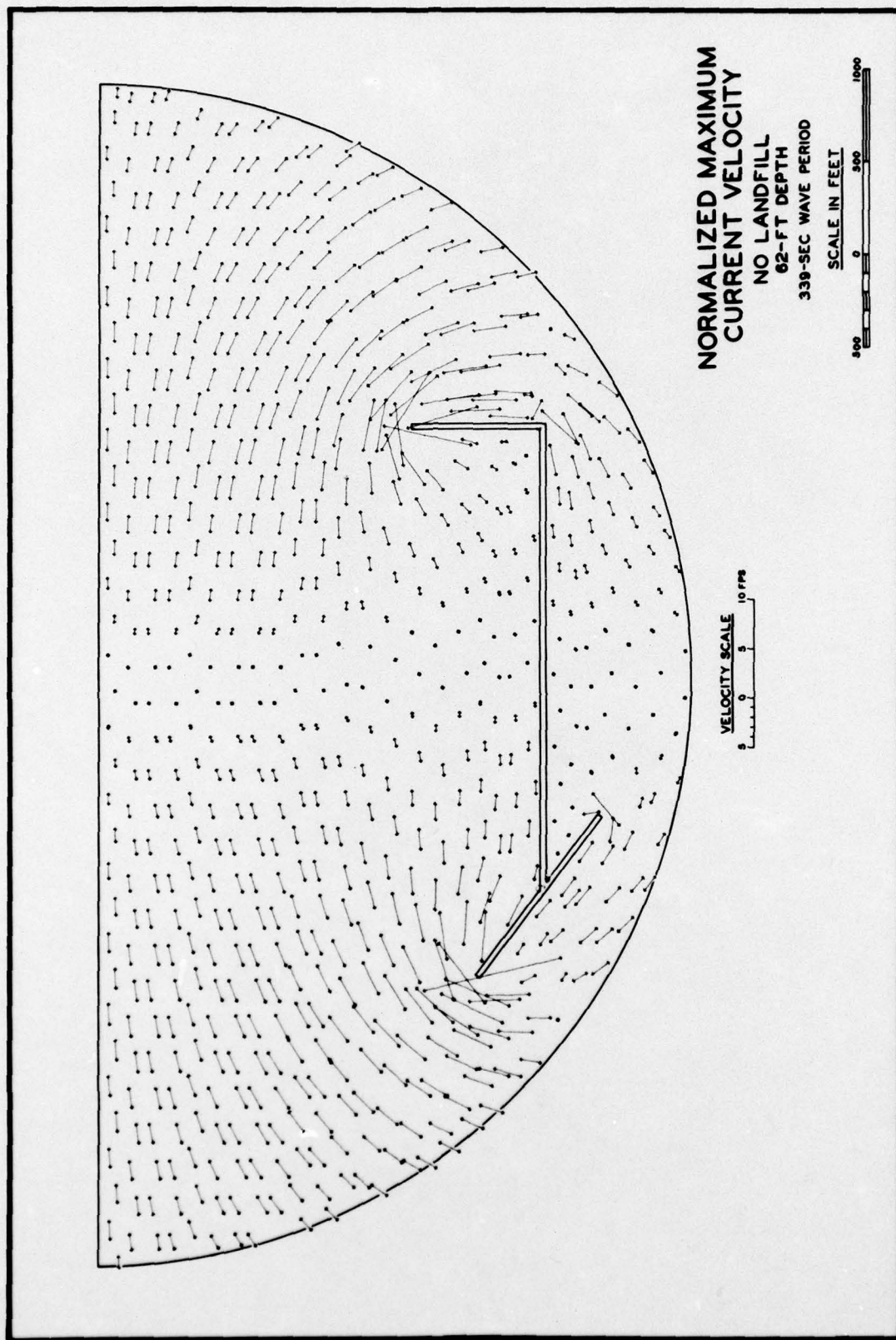
NO LANDFILL
62-FT DEPTH
44-SEC WAVE PERIOD

SCALE IN FEET

VELOCITY SCALE

PLATE 12

NORMALIZED MAXIMUM
CURRENT VELOCITY
NO LANDFILL
62-FT DEPTH
90-SEC WAVE PERIOD
SCALE IN FEET

VELOCITY SCALE

PLATE 13

NORMALIZED MAXIMUM
CURRENT VELOCITY
NO LANDFILL
62-FT DEPTH
124-SEC WAVE PERIOD
SCALE IN FEET

VELOCITY SCALE

PLATE 14

NORMALIZED MAXIMUM
CURRENT VELOCITY
NO LANDFILL
82-FT DEPTH
339-SEC WAVE PERIOD

SCALE IN FEET

500   0   500   1000

VELOCITY SCALE

5   0   5   10 FPS

PLATE 15

APPENDIX A: NOTATION

a   Boundary of region A

A   Area of region inside a harbor

g   Acceleration due to gravity, 32.2 ft/sec^2

h   Water depth, ft

H_n Hankel function of the first kind of order n

i   Imaginary number

k   Wave number, ft^{-1}

n   Integer

n_a Unit vector normal to boundary a

r   Spherical coordinate, ft

t   Time, sec

u   Velocity in x-direction, ft/sec

v   Velocity in y-direction, ft/sec

w   Angular velocity, radians/sec

x   Cartesian coordinate, ft

y   Cartesian coordinate, ft

α_n Unknown coefficient

∇ Gradient, ft^{-1}

η Wave amplitude, ft

θ Spherical coordinate, degrees

ϕ Total velocity potential, ft^2/sec

ϕ_a Total velocity potential evaluated on boundary a , ft^2/sec

ϕ_I Incident velocity potential, ft^2/sec

ϕ_R Far field velocity potential, ft^2/sec

ϕ_s Scattered velocity potential, ft^2/sec